Name:________________________

Math 3163 Section 01 | *Practice Exam 2* | *November 19, 2019*

Follow the instructions for each question and show enough of your work so that I can follow your thought process. If I can't read your work, answer or there is no justification to a solution, you will receive little or no credit!

1. Let G be an Abelian group of order $|G| = 16$. Suppose there are elements $a, b \in G$ such that $|a| = |b| = 4$ and $a^2 \neq b^2$. Determine the isomorphism class of G.

2. Let $p_1, p_2, \ldots, p_n$ be distinct primes and G an Abelian group such that $|G| = p_1 \cdots p_n$. What are the possible groups G is isomorphic to?

3. Let $S = \{a + bi : a, b \in \mathbb{Z}, b \text{ even}\}$. Show that S is a subring of $\mathbb{Z}[i]$, but not an ideal of $\mathbb{Z}[i]$.

4. Let R be a commutative ring and let A, B be ideals of R such that $A = \langle a \rangle$ and $B = \langle b \rangle$. Define $I = \langle ab \rangle = \{r(ab) : r \in R\}$. Show that I is an ideal of R.

5. Show that $\mathbb{R}[x]/\langle x^2 + 1 \rangle$ is a field.

6. Let $I = \langle 3+i \rangle$. It is known that I is an ideal of $\mathbb{Z}[i]$. Prove or disprove that it is a prime ideal. Can it be a maximal ideal?

7. Let R be the following ring:

$$R = \left\{ A = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : a, b, c \in \mathbb{Z} \right\}$$

Define the map $\varphi : R \to \mathbb{Z}$ via $\varphi(A) = a$. Show φ is a ring homomorphism. Determine the kernel of φ.

8. Let R be a commutative ring of prime characteristic p. Show that the map $x \mapsto x^p$ is a ring homomorphism from R to itself.

9. Let $f(x) \in \mathbb{R}[x]$. Suppose that $f(a) = 0$ and $f'(a) = 0$, here $f'(x)$ is the derivative of $f(x)$. Show that $(x - a)^2$ divides $f(x)$.